

WORKSHOP ON COVARIATION 2025

Planning

July 9th, Wednesday – Room 2.24		
14:00 – 14:15	Committee	Introduction
14:15 – 15:05	Osama Swidan	Hands-on workshop
15:10 – 15:30		Research presentation
15:30 – 15:55		Discussion
15:55 – 16:25	Coffee break	
16:30 – 17:20	Claudine Margolis	Hands-on workshop
17:25 – 17:45		Research presentation
17:45 – 18:10		Discussion
19:30	Free dinner	

July 10th, Thursday – Room 2.24		
08:50 – 09:00		Welcoming
09:00 – 09:50	Myrto Karavakou	Hands-on workshop
09:55 – 10:15		Research presentation
10:15 – 10:40		Discussion
10:40 – 11:10	Coffee break	
11:15 – 12:05	Eugenia Taranto & Sara Bagossi	Hands-on workshop
12:10 – 12:30		Research presentation
12:30 – 12:55		Discussion
13:00 – 14:30	Free lunch	
14:30 – 15:20	Hang Wei	Hands-on workshop
15:25 – 15:45		Research presentation
15:45 – 16:10		Discussion
16:10 – 16:25	Committee	Conclusion and greetings
16:30 – 17:00	Coffee break	
17:15	Walk and talk	
19:30	Free dinner	

Abstracts

Osama Swidan (Ben-Gurion University of the Negev)

Orders of covariational reasoning

Hands-on workshop

In this workshop, participants will explore three orders of covariational reasoning through a blend of theoretical discussion and dynamic, hands-on tasks. The session will be highly interactive, encouraging participants to solve problems and share their reasoning in real-time. Through this collaborative exploration, we will deepen our collective understanding of covariational reasoning and its developmental progression.

Research presentation

After introducing the three orders of covariational reasoning, we will present three research studies that examine students' reasoning processes in various contexts. In the first study, we will explore the emergence of second-order covariational reasoning as students engage in a physical simulation of a ball rolling down an inclined plane. The second study will focus on students' gaze patterns during mathematical tasks, using eye-tracking technology to investigate indicators of second-order covariational reasoning. Finally, we will examine students' third-order covariational reasoning as they solve a task involving the relationship between a function and its antiderivative.

Claudine Margolis (School of Education, University of Michigan)

Linking Digital Task Design to Covariational Reasoning: Sketch Interpretation Tasks

Hands-on workshop

In this workshop, participants will explore sketch interpretation tasks from three contexts: the bottle problem, distance-time, and the faucet task. In sketch interpretation tasks, students interact with an animated situation, sketch the relationship between quantities from the animated situation, and then view the interpretation of their sketch in the context of the animation. If you're able, please bring a laptop or tablet to engage with the tasks.

Research presentation

Sketch interpretation tasks provide a novel form of feedback in response to students' sketching activity. To investigate the role of task design choices on students' covariational reasoning, I designed a series of animation-based sketch interpretation tasks that varied along two key dimensions: numeric vs. non-numeric and well-defined vs. open. In this talk, I'll present findings from a study of undergraduate Calculus 1 students' covariational reasoning on sketch interpretation tasks.

Myrto Karavakou (National and Kapodistrian University of Athens)

Aesthetically guided covariational reasoning through creating digital dancing animations

Hands-on workshop

In this hands-on workshop, participants will engage in activities where covariation is connected to artistic expression in dancing movement. They will adjust and use trigonometric-periodic functions in order to create animations that align with musical rhythms, through interacting with dynamic and programming-based digital environments. The session will conclude with an open discussion on how such environments can foster embodied, creative, and inclusive covariational reasoning based on participants' personal experience.

Research presentation

In this presentation, I will share insights from my PhD research exploring how 11th grade students engaged in aesthetically guided covariational reasoning when designing and evaluating animations using periodic functions in digital environments. Students became choreographers of dynamic figural models and shaped mathematical meaning through aesthetic experiences. Their reasoning processes were analysed as patterns of continuously re-formed choreographic ideas and evaluative criteria regarding graphical attributes of periodic functions under use, such as period, amplitude, monotony and slope. This study discusses how such technological and aesthetic environments facilitate multimodal and creative engagement in covariational reasoning on periodic functions, highlighting both theoretical perspectives and practical implications.

Hang Wei (Utrecht University)

Covariation in action: Syncing hands and minds in digital play

Hands-on workshop

Join this hands-on workshop to sync your hands and mind in a digital adventure! Using the Numworx platform on your phone or tablet, you'll manipulate functions and experience covariation through bimanual movements. This fun session reveals how embodied design sparks covariational reasoning. Bring a device and get ready to play and learn!

Research presentation

I'll unpack a hand-tracking study with 76 Grade 9 students, exploring how bimanual movement in a digital-embodied setting boosts covariational reasoning (CR). Guided by Thompson and Carlson's (2017) taxonomy, we operationalized a CR framework for bimanual movement, mapping levels from uncoordinated wiggles to smooth synergy. I'll show some representative hand-tracking cases, highlighting features like trace continuity, coordination, and feedback students got from the digital environment. Plus, interview snippets will showcase how perception-action loops, attentional anchors, and real-time feedback facilitated students' grasp of CR.

Eugenia Taranto & Sara Bagossi (Kore University of Enna; Free University of Bozen-Bolzano)

Exploring covariation with the physical and the digital Tracer

Hands-on workshop

Working in small groups, participants will be involved in a series of tasks to engage in covariational reasoning using a duo of artifacts. This duo consists of: 1) a *physical Tracer*, made up of a wooden panel with two perpendicular drawer sliders, each fitted with a knob to control a rigid bar. By manipulating these knobs, users can move the intersection point of the bars to draw or trace lines on a paper sheet placed on the panel; 2) a *digital Tracer*, made with the free software Geometric Constructor, which reproduces some of the characteristics of the physical Tracer and in addition allows the trace of the path that the intersection point takes to be evident. Participants will be requested to follow some given traces by using the two artifacts and to reflect on the covariational conceptualization that such task design might prompt.

Research presentation

In this presentation, we will explore the mathematical conceptualization of covariation that can emerge in students while using a duo of artifacts consisting of a physical and a digital Tracer. Such artifacts enable the design of tasks that emphasize bodily engagement and multimodal experiences. After introducing some design principles for an embodied learning environment, we will interpret the task design with the duo of artifacts according to those principles. Adopting the framework of embodied instrumentation, we will examine the intentionality, bodily experience, stabilized behaviors, and conceptualization of peers working together in order to follow some given traces using the duo of Tracers. Such analysis will bring to the fore the emergence of action-perception loops while peers work together so forming an intercorporeal functional dynamic system.